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DEFINING KNOWLEDGE: GETTIER-LIKE SCENARIOS

Abstract: *In this paper we analyse the requirements for a redefinition of knowledge that would block all the Gettier-like counterexamples raised against the traditional definition of knowledge. For this purpose we provide a classification of all the possible Gettier-like counterexamples. We claim that this classification is exhaustive because it lists all the logical representations of the Gettier-like counterexamples. This enables us to define a Gettier-like counterexample and suggests a definition of knowledge that would not allow counterexamples of such a sort. Our approach is proof-theoretical, i.e. our formal representation of a Gettier-like counterexample is proof-theoretical and, hence, our definition is in line with it.*

Keywords: *knowledge, belief, Gettier-like counterexample, logic*

1.Introduction

It the dialogues Theaetetus and Meno many philosophers have read Plato as giving a definition of knowledge as true, justified belief, often referred to as a traditional definition of knowledge.[1] In his famous article [1963], Edmund Gettier provides a counterexample to this definition, i.e. he shows that one can have a justified true belief that φ, but still not know that φ. In the course of years many attempts were made to either strengthen Plato's traditional definition of knowledge or to redefine knowledge completely in such a way that it can account for Gettier's counterexamples. On the other hand, some new Gettier-like scenarios have been proposed. We will refer to this work in epistemology as the Gettier program.

In this paper we will first have a look at Gettier's original counterexamples in order to define what a Gettier counterexample in general is. We will show that

1 See Plato, Theaetetus and Meno, translated by Benjamin Jowett (1871), available at Project Gutenberg: www.gutenberg.org.

it is possible to provide Gettier counterexamples corresponding to each logical connective. We will also show that there are Gettier counterexamples that are not connected to properties of logical connectives and that seem to belong to a different class. Having done this, we will try to classify different kinds of Gettier counterexamples. In our opinion, distinguishing between different Gettier examples helps in detecting the defects of the JTB definition.[2]

Finally, we will provide our definition of knowledge. We will first give some general observations drawn from the previous findings and then based on this give a new definition of knowledge that is an extension of JTB by adding a fourth condition. What is important in this solution is that we think that whereas JTB only takes into account one kind of justification, the definition of knowledge should incorporate two kinds of justification. Our idea will be more like a suggestion than a solution, and is open to further developing.

2. Motivation

In philosophy understanding what knowledge really is, is important in order to reply to radical sceptics. Radical sceptics doubt the existence of the external world. The debate about scepticism started in Ancient Greece and is still lively. Another important philosophical theme where knowledge plays a central role, closely connected to the previous one, is the eternal quest for certainty. Knowledge is something certain. When it comes to certainty, we have asymmetries between different types of knowledge. On the one hand, there is analytic knowledge, which is the knowledge acquired only on the basis of analysing concepts. On the other hand, there is synthetic knowledge that brings something new to the original concepts. Following Jaakko Hintikka, one can understand analytic propositions as natural deduction trees with all the premises on the leaves discharged.[3] Rene Descartes in [1996] was the first one to point out that we cannot doubt one proposition "*Cogito, ergo sum*", which means that we cannot doubt our own existence. Nevertheless, all the beliefs we have about the external world have a different status.

It should be noted that the distinction analytic/synthetic has been attacked by Willard Van Orman Quine in his famous article *Two dogmas of empiricism* (1953). Nevertheless, we will assume truths of logic together with Descartes' self-referential proposition to be analytic and all the others truths to be synthetic. For the purposes of our paper this simplification does not give a problem. In this paper we will focus on answering the question what the necessary requirements are for a person to have knowledge of synthetic propositions. We will also have in mind natural deduction derivations, but from the premises, i.e. with some non-discharged assumptions on the leaves.

When studying epistemic logic, it is important to understand key notions from epistemology, such as knowledge and belief. The topics we are treating

2 Below we will abbreviate the three conditions mentioned here as JTB (Justified True Belief).

3 See Hintikka, J. (1965) Are logical truths analytic? *Philosophical Review*, Vol. 74, pp. 178–203.

in this paper are in the very core of epistemology. We are searching for a definition of knowledge. This topic is relevant for all attempts to formalise the notion of knowledge. Concretely, before a formal account of the Gettier cases is possible, we need a good conceptual understanding of what they are. Our analysis should provide both better understanding of philosophical concept of knowledge and give the foundational insights for further formalisations of this concept.

3. What is a Gettier counterexample?

3.1. Justified True Belief (JTB) not sufficient

In his short paper 'Is Justified True Belief Knowledge' (1963) Edmund Gettier shows that the following three conditions, which constitute the traditional definition of knowledge, are not sufficient conditions for an agent S to know a proposition:

(i) P is true.

(ii) S believes that P.

(iii) S is justified in believing that P.

Gettier does this by providing two counterexamples, in which an agent satisfies all these three conditions but does not have knowledge. We will here reconstruct one of his examples. Suppose Mr. Smith and Mr. Jones both applied for a job. Suppose further that Mr. Smith has strong evidence that not he but Mr. Jones will get the job (the boss told him so). Furthermore, Mr. Smith has strong evidence that there are ten coins in Mr. Jones' pocket (Mr. Smith counted them). Therefore Mr. Smith has evidence for (a) and (b):

(a) Mr. Jones will get the job.

(b) Mr. Jones has ten coins in his pocket.

Propositions (a) and (b) together entail proposition (c):

(c) The man who will get the job has ten coins in his pocket.

Now suppose Mr. Smith sees this entailment. Therefore, he believes (c). We observe that Mr. Smith is justified in believing (c). Now suppose that in fact, unbeknownst to Mr. Smith, Mr. Smith himself will get the job, not Mr. Jones. By coincidence, Mr. Smith himself, also unbeknownst to himself, has ten coins in his pocket as well. Now the following three conditions are satisfied:

(i) (c) is true.

(ii) Mr. Smith believes that (c).

(iii) Mr. Smith is justified in believing that (c).

Nevertheless, clearly Mr. Smith does not *know* (c). Therefore, it is not knowledge. Hence, the three conditions of JTB are not sufficient for knowledge.

3.2. Definition of a Gettier counterexample in general

Since Gettier's paper many other counterexamples to the traditional notion of knowledge as true justified belief have been published. In all these examples an agent has true, justified belief, but does not have knowledge. We will call all such examples 'Gettier counterexamples/Gettier cases' from here on, according to the usual practice in the literature. We will show that it is possible to construct such examples using different logical connectives and we will also see how Gettier counterexamples can appear on a meta-level.

4. Logical Connectives in Gettier Counterexamples

It is thought that Gettier counterexamples to the traditional definition of knowledge are connected with the special feature of disjunction, i.e. the fact that if only one disjunct is true, the proposition with a disjunction as a main connective is also true. In this section we will provide a Gettier-like counterexample for all intuitionistic logical connectives.[4]

4.1. Disjunction case

A disjunction case is the obvious one, in [1963] Gettier himself proposed one counterexample of this type. An agent believes that her friend Jones owns a Ford. She has seen him driving a Ford and she has even been in the car with him. The agent also has another friend, Brown. She is totally ignorant about Brown's whereabouts. Nevertheless, because of the property of disjunction she is justified to form a belief that *Either Jones owns a Ford, or Brown is in Boston.* This turns out to be true justified belief. Still, in fact Jones does not own a Ford, but by coincidence Brown happens to be in Boston and this is what makes her belief true. We should observe that these two derivations are not the same:

$$\frac{A}{A \vee B} \qquad \neq \qquad \frac{B}{A \vee B}$$

Proposition A stands for "John owns a Ford", while B stands for "Brown is in Boston".

The first derivation stands for the manner in which the agent actually formed her belief, while the other derivation represents the actual justification of the sentence. From a proof-theoretic perspective these two derivations are different. The left derivation is valid (correct) but not sound (because of the false premiss), whereas the right derivation is sound. On the left side we have a derivation corresponding to the agent's justification, while on the right side we have the 'actual' derivation that makes sentence A $\vee$ B ("Either John owns a Ford or Brown is in Boston") true. From the Gettier counterexamples we can learn that in order for an agent to have knowledge she must construct the right

4 Everything that we will show for intuitionistic logical connectives also holds true for classical connectives.

justification. One of the proposals for this was that there should be no false lemmas in her justification. In the case of disjunction as presented above this restriction is sufficient to block the Gettier counterexample.

4.2 Implication case

When it comes to implication we can formulate Gettier-like counterexamples in the following manner. Propositions with the implication as a main connective can be true because of the falsehood of the antecedent or because of the truthfulness of the consequent. This is exactly how we form the Gettier-like properties. Therefore, we can distinguish two cases of Gettier counterexamples that involve implication.

Implication case (a) For instance, an agent heard on the news that on Tuesday there will be the coronation of the new Roman pope, namely Francis. Therefore, she forms a belief that *If today is Tuesday, then Francis is the Roman pope.* This turns out to be a true belief, because Francis is indeed the new Roman pope, but his coronation was on Monday. Once again, the agent has formed justified true belief that is not knowledge. We can see the difference between the two derivations:

$$\dfrac{\dfrac{\begin{matrix} \mathrm{P} & [A]^1 \\ \vdots & \vdots \end{matrix}}{B}}{A \to B}\,1 \qquad \neq \qquad \dfrac{B}{A \to B}$$

Where A stands for *Today is Tuesday*, B stands for *Francis is the new pope* and P for the news *On Tuesday there will be a coronation of the new Roman pope, Francis.* On the left we see the general pattern of the agent's justification that A→B.

The agent heard the news that the coronation of the new pope Francis will take place on Tuesday. The belief that she forms that A→B (*If it is Tuesday, then Francis is the new pope*) is justified by the news that she heard. We see this in the derivation on the left side. However, this derivation is not sound because the news P the agent heard was false. On the right side, we see the derivation of A→B based only on the fact that Francis is the pope, which is a sound derivation. Clearly, the objective justification of the proposition A→B does not match the one of the agent.

Implication case (b) There is also another type of Gettier counterexamples that is connected with implication. According to our example, we can present it as follows. The agent's belief that *If today is Tuesday, then Francis is the Roman pope* is justified because of the news she heard that the coronation of the pope will be on Tuesday. Also, this belief turns out to be true because it is already Wednesday. Nevertheless, the news that the agent heard was false, so today is not Tuesday. We can distinguish between two different derivations: Where A stands for *Today is Tuesday*, B for *Francis is the Roman pope* and P stands for *On Tuesday there will be the coronation of the new Roman pope, Francis*. It is important to emphasise that P≠¬A. On the left side we have the way how the agent came to believe A→B (the agent's justification) and on the right side we have actual

$$\dfrac{\dfrac{\begin{matrix} \mathrm{P} & [A]^1 \\ \vdots & \vdots \end{matrix}}{B}}{A \to B}\,1 \qquad \neq \qquad \dfrac{\dfrac{\dfrac{\neg A \qquad [A]^1}{\bot}}{B}}{A \to B}\,1$$

reasoning that makes A→B true. Again, on the left side we have a premise P that is false, so the left argument that A→B is valid, but not sound.

4.3 Negation case

We can think of ¬A as abbreviation of A→⊥. Therefore, we only need to impose some restrictions on our implication analysis. Intuitionistic ***Negation case (a)*** is impossible. Since we cannot consider inconsistent systems, we can never be in the situation where A→⊥ is true because of the truthfulness of the succedent. Since in this case the succedent is falsehood, it can never be true.

Negation case (b) Suppose the agent believes *It is Tuesday* and she heard on the weather forecast that it will rain on Tuesday and forms a belief *If it was Tuesday then it would rain*. Next, the agent sees through the window that It is not raining, so she rejects her earlier belief that it is Tuesday. She forms a belief *It is not Tuesday*. This turns out to be justified true belief, but in fact it was raining, but the agent failed to see it from her window. The proposition *It is not Tuesday* is still true, because it is not Tuesday and not because it is raining. Analogously to *implication case (b)* the antecedens, *It is Tuesday* is false and this is the reason why the belief is true, not because of the other premises. We can distinguish between two derivations.

$$\cfrac{\cfrac{\begin{matrix}\mathcal{P} & [A]^1 \\ \vdots & \vdots\end{matrix}}{\bot}}{\neg A}\,1 \qquad \neq \qquad \neg A$$

Where on the left we see the agent's subjective justification, and the objective justification that ¬A on the other side.

$$\cfrac{\cfrac{\cfrac{[A]^1 \qquad A \to B}{B} \qquad \neg B}{\bot}}{\neg A}\,1$$

The precise extended version of the agent's proof looks like the following: Where A stands for *It is Tuesday*. Propositional letter B stands for It is raining and A→B stands for *If it was Tuesday then it would rain*.

Another remark about negation is that from falsehood one can derive anything. If we do not want to allow this principle in our justification derivation, then we need to abolish the *ex falso quodlibe*t rule. Then we will be using minimal implication (not intuitionistic).[5]

4.4 Conjunction case

It is difficult to construct Gettier counterexamples that involve conjunction as a main connective. The counterexamples that we can think of are based on the property of conjunction that it is commutative, as will be illustrated by the example below. Nevertheless, not every conjunction is commutative. Non-

5 Minimal logic is obtained from intuitionistic by abolishing the *ex falso quodlibet* rule.

commutative conjunction is the most primitive conjunction. It is hard for us to imagine a counterexample to the JTB-definition of knowledge of a Gettier type that would involve a non-commutative conjunction as the main connective in the conclusion. It would be interesting for further research to understand why this is the case, keeping in mind that both knowledge and belief operator distribute over this type of conjunction.

Since intuitionistic (and classical) conjunction are commutative we give the following counterexample. An agent sees a poodle dog and a sheep in front of her, so she believes that there are a poodle and a sheep in front of her. This happens to be a true justified belief. Nevertheless, as a matter of fact, she mixed them up. What she believed to be a sheep actually is a poodle dog and what she believed to be a dog is actually a sheep. Are we ready to say that she has a knowledge that there are poodle and sheep in front of her? Observe these two derivations:

$$\frac{A \qquad B}{A \wedge B} \qquad \neq \qquad \dfrac{\dfrac{B \qquad A}{B \wedge A}}{A \wedge B}\,Commutativity$$

Propositional letter A stands for "There is a sheep in front of me", while B stands for "There is a poodle dog in front of me". On the left side, we see how the agent comes to believe "There are a sheep and a dog in front of me", while on the right side we see the actual justification that makes proposition "There are the sheep and the dog" true. We see that this proposition is true because the connective *and* in natural language is commutative, but this is not the case in the logic of justification. There we have to take care of the manner how we introduce $\wedge$.

When analysing proof-identity criteria, i.e. identity between derivations, there are two main propositions. One comes from natural deduction and that is the fact that normalisation divides proofs, in our case derivations, into equivalence classes. According to this criterion the derivations above are the same. Nevertheless, according to the other more refined proof-identity criterion two derivations from above are different. In the left derivation according to the fine-grained proof-identity criterion the premisses are false because they need to be commuted to be true.

4.5 Existential quantifier case

The agent shares the office with two colleagues, Rachel and Susan. The agent believes that Rachel owns a Ferrari. She saw Rachel driving it and parking it in front of the office. Now, the agent forms a justified true belief that someone in her office owns a Ferrari. Nevertheless, the truthfulness of this belief is due to the fact that Susan owns a Ferrari, not Rachel. We can formalise this by having as domain D = {r, s, a}, where r stands for Rachel, s for Susan and a for our agent. We can observe two different derivations:

$$\frac{Fr}{\exists x Fx} \qquad \neq \qquad \frac{Fs}{\exists x Fx}$$

For each introduction of the existential quantifier we have a different witness. On the left side that is Rachel and on the right side that is Susan. Again one is a derivation of how the agent formed her belief and the other is the actual reason why this belief is true. Only the derivation on the right is sound.

4.6. Universal quantifier case

We will now give a Gettier counterexample that involves special properties of the universal quantifier. Imagine that in a classroom all children with red hair have iPhones. The agent thought that she saw all the children with red hair holding iPhones in their hands. As it turns out, they were all wearing wigs and as a matter of fact, there are no children with red hair in the classroom. The proposition "All children with red hair have iPhones" is still true because there are no children with red hair in that school. It is also justified and believed by the agent.

Now imagine a slightly different scenario. Some of the children that had red hair were wearing wigs of a different colour, but they still possess iPhones. Again, the proposition "All children in the classroom with red hair have iPhones" is true, justified and believed by the agent. Also different overlaps are possible, like some children with different hair colour were wearing red wigs, but in fact they only played with borrowed iPhones, or the combination of these two.

4.7. Towards a solution

From the Gettier counterexamples we can learn that having knowledge means constructing the right justification. One of the proposals for an improvement of JTB was to add a no-false lemma restriction. The no-false lemma restriction means that justification cannot be based on any lemma that is false. On the other hand, it is also possible to add new conditions for knowledge that will guarantee that the agent has the right justification and this will turn out to be necessary. We will show that simply strengthening the definition of justification cannot solve the problem. If the justification of the agent matches the objective justification of a proposition, then we can safely say, from an external and omniscient point of view, that she has knowledge. Objective justification is the one that from an objective perspective justifies the proposition. We will come back to this in our own proposal.

5. Other Gettier counterexamples

In the previous section we saw how for each logical connective Gettier counterexamples can be constructed. In fact, all these counterexamples seem to share a common feature: there is a false premiss in the agent's reasoning, a false lemma.

However, there is another way of constructing Gettier counterexamples. This is by incorporating in the example information that makes the agent's justification unreliable from an omniscient point of view. To be more precise, we mean the following by this: the agent has justified true belief in a proposition p. The justification is based on a set of premisses Γ. However, there is another true proposition q implying that the premisses in Γ are not sufficient evidence in this context that p is true. The agent does not know that q. Nevertheless, the agent is justified in believing that p based on the evidence in Γ, because Γ is in absence of special circumstances —and q is a special circumstance— such that it justifies p. Moreover, all premisses in Γ are true, which makes these kinds of scenarios crucially different from those in the previous section. As a concrete example of this, consider the following scenario:

Suppose there is a country in which almost all barns are fake. These fake barns look as if they are real barns but in fact they are just facades (but indistinguishable from real barns). Now suppose a tourist who does not know that almost all barns in the country are fake is visiting this country. This tourist is standing in front of a barn at location X in this country. As it happens, this barn is one of the few in the country that is real. The tourist, seeing this barn in front of her, comes to believe the following proposition p: 'The thing standing at location X is a barn'.

Now the agent has justified true belief in p based on the set Γ, which here consists of perceptual evidence (the agent sees a barn), which would in normal circumstances suffice to conclude that there is a real barn. Γ does not include false evidence, because the agent actually sees a real barn. However, the agent is unaware of the proposition q stating that almost all barns in this country are fake. Therefore the agent fails to have knowledge: to have knowledge, she should know q and she should know that the very barn she sees is not fake. In other words: in order to have knowledge the agent should have a justification to exclude a salient contextual alternative related to q, namely that the barn she sees is fake. Intuitively, the reason that the agent has no knowledge is that the truth of her belief seems to be a mere coincidence: had the agent been standing before a fake barn, she had also believed there was a real barn in front of her.

6.Classification

6.1. Twofold classification

In this part we will discuss a twofold classification of the Gettier counterexamples. First we will discuss a 'naive' way of classifying the examples into a false lemma and a no false lemma class. Then we will give an objection to the description of these classes and provide an alternative, more rigorous classification. The classification should give more insight into how new Gettier counterexamples can be constructed. The classification is a priori. All the possible Gettier counterexamples should be covered in this classification. This means that we need to account for the Gettier counterexamples that involve all the logical connectives and Gettier counterexamples of the other type described above. We have grounds to think that all possible Gettier counterexamples belong to one of these classes. First of all Gettier counterexamples involve some logical reasoning, for instance they are not about perceptual illusions. We have examined all the possibilities of constructing Gettier counterexamples with different logical connectives and their corresponding rules of introduction. The only other possibility to construct a Gettier counterexample would need to involve higher order reasoning. This is precisely what we have in the other type described above. The problem there is not truthfulness of a premiss itself, but whether the premises are justified or sufficient for deriving the conclusion.

In any case we did not find a precise classification of Gettier examples in the literature, whereas we think it is useful to make one because it gives precise insight into what exactly is lacking in JTB. The different classes of Gettier examples give different answers to this question.

6.1.1. Cases with false lemma

Gettier's own examples belong to this class. The same goes for the examples in the chapter on Gettier counterexamples and logical connectives. In these examples *the justification of an agent's belief depends on a false assumption*. In the example from the Gettier paper reconstructed above (the Mr. Smith-Mr. Jones-case) this is the false proposition that Mr. Jones will get the job (proposition (a) in the above). From this proposition and the true proposition (b), Mr. Smith derives proposition (c), which happens to be true. Therefore Mr. Smith's derives his true belief from one or more false premisses that he is justified to believe (a false *lemma*).

6.1.2. Cases without false lemma

In Gettier counterexamples without false lemma there is *no false proposition that plays a role in the agent's justification, the agent's belief is not derived from false lemma's*. These counterexamples are the ones described above in the chapter on other Gettier examples.

Consider for example the barn case mentioned there, where the agent concludes that she sees a barn standing in front of her. There seems to be no false evidence involved in the justification there, whence there is no false lemma in the justification.

6.2. Objection to the above classification

One could object to the above classification and argue that in fact also the no false lemma Gettier cases do have a false lemma, albeit implicit. In the barn example the agent implicitly and falsely assumes that the following proposition *(a)* is true:

(a) If something in this country looks like a barn, it is a barn in most cases.

If the agent did not (implicitly) assume *(a)*, it would not of course imply that the specific barn she perceives at location X is fake, but it would mean her justification that she sees a real barn would be blocked because normal perception is not reliable enough if *(a)* is false. In other words: if the agent did not believe *(a)*,[6] mere perception of something that looks like a barn is no justification that something actually is a barn. Mere perception is a justification if the agent does believe *(a)*, which in fact she did in the example. We could think of the *negations*

6 Or maybe: if the agent believed *(a)* was false.

of these implicit assumptions as information that would, if given to the agent, *defeat* the agent's belief.

However, we think it is not reasonable to see the hidden/implicit false premisses in the second group of examples as actual premisses in the justification of the agent. We will give a different, more accurate classification below.

6.3. Another way of defining the classification

One may argue that there is after all a difference between false lemma and no false lemma Gettier cases, but this needs to be defined in a different way than above. In both classes of counterexamples the agent is not informed enough to have knowledge. In the false lemma cases this is due to a very primary reason: *the evidence that justifies the agent's belief is false*. With 'evidence' we hereby mean that the propositions (lemmas) from which the agent derives her belief. What the agent believes is true not because of what the evidence indicates, but because of something else. In the no false lemma cases on the other hand the evidence as such is not false. However, *the evidence is unreliable due to the context*. The evidence is not sufficient for constructing a justification that yields knowledge, because the agent would not be justified in believing the proposition any more if she knew what justification power of the evidence is. In these Gettier cases the evidence is in 'normal' circumstances good enough to make the agent believe something, but due to special, context-dependent circumstances the evidence is not reliable.

Now we can give the following classification:

1. The agent has a justified true belief but does not have knowledge since part of the evidence that justifies the belief is false.
2. The agent has a justified true belief but does not have knowledge since the evidence that justifies the belief is not sufficient given that there are things the agent does not know.

There is a clear difference between examples of class 1 and 2. Group 1 consists of Gettier counterexamples concerning different logical connectives. Group 2 consists of *meta-level* Gettier counterexamples. The intuition one has to have about whether there is knowledge are less clear when it comes to the cases of class 2. This is because the information the agent lacks in class 2 cases is of a 'higher order' and in less basic (and hence possibly less important) than the information that agents in case 1 lack.

7. Proposition

7.1. Knowledge and conditional belief

If we consider knowledge based on conditional belief, we might end up with the following situation. For an agent the following proposition φ might be true: *I saw someone looking like Tom stealing a book.* Conditionally on this evidence, the agent can form a belief ψ: *Tom Grabbit stole a book*. In this case the agent has a conditional belief that ψ based on φ, formally written as B^{φ}_{ψ}. This is a justified true belief, with no false lemmas in the premisses. Even more strongly, the agent has knowledge of the premiss φ obtained by introspection.

Now, imagine that the agent finds out that proposition κ: *Tom Grabbit has a twin brother who is inclined to stealing books* is true. After hearing this new true information the agent does not believe ψ: *Tom Grabbit stole a book* any more, i.e. $\neg B^{\varphi \wedge \kappa}_{\psi}$. Nevertheless, ψ is the case. Before learning κ the agent had justified, true belief, but are we inclined to state that she knew ψ?[7]

If our intuition says that ψ is not knowledge, then we are dealing with a Gettier counterexample. According to our classification this counterexample belongs to group 2. Proposition ψ is a justified true belief, based on true lemma φ. Since the no false lemma requirement does not block this counterexample, an additional requirement in the definition of knowledge is needed. For instance, Baltag, Renne and Smets (2012) propose the following definition of knowledge, which they call defeasible.[8] Something is defeasible knowledge *iff* it is believed after learning any new true information. Defeasible knowledge in this sense should be understood as empirical knowledge. In our example, after learning true proposition κ the agent is not any more willing to believe in ψ and, because of this, she does not have a defeasible knowledge that ψ. Furthermore, this proposal can deal with sceptic scenarios, because after learning that *It is possible that we are systematically deceived by an evil demon* we still believe that ψ. Differently phrased, sceptic worlds are too far from the actual one and do not affect our beliefs.

We would like to point out the following. Learning that κ: *Tom Grabbit has a twin brother who is inclined to stealing books*, is different from learning θ: *Tom Grabbit has a twin brother*. After learning κ the agent will probably reject her belief that ψ. Nevertheless after learning θ, the agent will revise her degree of belief in ψ, but she could still believe it. In this case she would have knowledge according to the defeasible knowledge proposal. Still in another situation the same evidence θ might affect the agent's reasoning differently. When people are more involved in the situation, or care more about its consequences, they rank same evidence differently. For instance, if our φ is: *I saw someone looking like Tom Grabbit killing a man* and the corresponding ψ: *Tom Grabbit killed a man*, then learning that θ: *Tom Grabbit has a twin brother* might suffice to make the agent doubt that Tom is guilty. In this case the agent might stop believing that ψ, therefore both according to JTB and the defeasible knowledge proposal ψ is not knowledge. What the defeasible knowledge proposal does not account for, is why in the first case the same evidence did not affect the agent's knowledge and in the second it did. We believe that which alternatives will be relevant for knowledge is a matter of pragmatics and can change depending on the situation.

7 Thanks to Professor Alexandru Baltag for this refinement of the 'Nogot/Grabbit' example, which he presented to us in a meeting on the 4th of April, 2013.

8 See (2012).

What we should observe is that conditional belief is not a monotonic operator. If the agent conditionally on φ believes ψ, i. e. B^{φ}_{ψ}, this does not mean that she will believe ψ conditionally on $\varphi \wedge \kappa$ (where κ is true). Our general approach can be refined depending on intuitions one has about knowledge. For instance we can put restrictions on the logical connectives we use and not allow the deduction theorem in order to abolish monotonicity. A similar technique of putting restrictions on the logical connectives in the case, can be employed if one wants to show that knowledge is not closed under implication. Still this is just a proposal in the sense that it sets a framework for defining knowledge and justification and not a developed theory yet.

7.2. Proposal

The general underlying structure of Gettier counterexamples can be analysed by pointing at the logical derivation based on which the agent obtains her knowledge. Firstly, Gettier does not talk about pure perceptual illusions, but about knowledge that is derived through certain inferences. What Gettier counterexamples show is that, even though the conclusion follows from the premise with which the agent operates, she still does not know the conclusion. This is because the truth of the conclusion is independent of the deduction that the agent makes. A sound argument is the one that is obtained from true premisses by valid deductions. In this spirit we can formulate our proposal: p_1: **"For knowledge to be justified, the agent has to obtain it from other knowledge by valid deductions".** In the example above we have seen that the agent might form a justified true belief that ψ based on knowledge that she saw someone like Tom Grabbit stealing a book. We see that this definition is not satisfactory to deal with all of the meta-type (i.e.class 2) Gettier counterexamples.[9] This leads us to a different proposal, p_2: "**The factual truthfulness of the premisses in the agent's justification is sufficient for the factual truthfulness of the conclusion in the agent's justification".** All of the counterexamples from this paper that involve logical connectives can be accounted for by this definition. What need be done, is to show how this definition can be incorporated into the definition of knowledge and how it accounts for meta-type Gettier counterexamples.

7.3. A new solution: a proof-theoretic approach

The solution we present here is inspired by our proof-theoretic representation of justification and all the previous considerations.

Definition 1. An agent *S* knows *p* if and only if:

- Conditions (i)-(iii) from JTB are satisfied.
- Condition (iv) is satisfied, where condition (iv) is: the subjective justification of the agent for *p* needs to match the objective justification for *p*.

9 Note that, if the true premise φ would be formulated differently, i.e. "I saw Tom Grabbit running out of the library with a book", the proposal p_1 in this case would be sufficient to block the counterexample. The agent would not have knowledge of the premiss.

We can define objective justification in the following manner.

Definition 2. An objective justification is a justification that:

1) is factual: the argument is sound, i.e. it is a valid deduction from true premisses;
2) takes into account relevant alternatives.

Where *relevant alternatives* are false propositions that would, if not excluded by the agent, render the agent's premisses insufficient for a justification of the conclusion from an omniscient point of view. If there are relevant alternatives for the agent's premisses, the agent needs to include the negation of these alternatives in her justification in order for her justification to coincide with the objective justification.

Part 1 is sufficient condition to abolish the first type of Gettier counterexamples as presented in the classification. While part 2 takes care of the second type of Gettier counterexamples. Let us consider how this works in the following examples.

Case 1

We will show how our definition deals with the Gettier counterexamples that belong to the first group in our classification. Remember Gettier's own example where the agent believes A:=*Jones owns a Ford*. From this she forms a belief $A \vee B$: *Either Jones owns a Ford, or Brown is in Boston,* even though she is totally ignorant about Brown's whereabouts. It turns out that Jones does not own a Ford. On the other hand, the belief that *Either Jones owns a Ford, or Brown is in Boston* is objectively justified because of B, the fact that Brown is in Boston. We can see why subjective justification on the left side does not match the objective justification on the right side:

$$\frac{A}{A \vee B} \quad \neq \quad \frac{B}{A \vee B}.$$

All classical Gettier counterexamples, those that belong to the first group of our classification, have the same shape. The inference one uses to form knowledge is valid, but not sound, because it uses at least one false premise. On the other hand, according to our definition, objective justification has to be sound.

Case 2

Now we will show how our solution deals with the Gettier counterexamples that are in the second part of our classification, i.e. with meta-level counterexamples. In the example in section 7.1 the agent has introspective knowledge that φ: *I saw someone looking like Tom stealing a book.* Conditionally on this evidence, the agent can form a belief ψ: *Tom Grabbit stole a book*. In this case the agent has a conditional belief that ψ based on φ, i.e. B^{φ}_{ψ}. Nevertheless, proposition κ: *Tom Grabbit has a twin brother who is inclined to stealing books* is true. This means that there is a relevant alternative of φ, namely τ: *Tom's twin brother stole the book.* According to our definition, objective justification of φ has to have $\neg\tau$ as one of the premisses, since ignorance of τ would render the agent's justification insufficient from an omniscient point of view. We have the following derivations: Where $\neg\tau$

$$\frac{\begin{matrix}\varphi\\ \vdots\end{matrix}}{\psi} \quad \neq \quad \frac{\begin{matrix}\varphi\\ \vdots\end{matrix} \qquad \begin{matrix}\neg\tau\\ \vdots\end{matrix}}{\psi}$$

is necessary for deriving ψ. On the left side we have the subjective justification of the agent and on the right side we have objective justification and we see that these two derivations do not match.

In addition to this, the definition of knowledge we propose can deal with the pragmatic contexts in which relevant alternatives differ. When a jury is considering convicting someone for murder, the set of premisses of objective justification is bigger than the set of premisses when considering a case of stealing a book from a library. Also, radical sceptic scenarios do not represent a relative alternative when forming every day belief, but can be taken seriously in philosophical discourse.

8. Further research

Further research could be done on identity criteria between subjective and objective justification, inspired by proof-identity.

subjective justification ↔ objective justification

One of the possibilities would be to adopt a normalisation-like criterion for the identity of derivations. If we think of the normalisation criterion for proof-identity then we can allow on the side of subjective justification non-normalised derivations, while on the side of objective justification there can only be normalised derivations by definition. This means that the agent is allowed to have repetitions in her reasoning if her argument is sound.

We can illustrate this by the following example. The agent can make the following detour in her reasoning. She knows that *If the street light is green she may pass*. Then she reasons 'if the light is indeed green then from this fact and my knowledge I indeed may pass' and then again she can draw a justified true belief (from the initial premise) that *If the street light is green she may pass.* Such subjective justification is formally represented on the left, while the objective justification (the one without detours in reasoning) is represented on the right.

$$\frac{A}{A \vee B} \quad \neq \quad \frac{B}{A \vee B}.$$

Note that in this case we would be keen to say that subjective justification matches objective one modulo certain identity criterion.

We have seen that it is difficult to construct Gettier counterexamples for some primitive connectives, like non-commutative conjunction. Our solution is different from Sergei Artemov's (2008). We do not give an account of the logic of objective justifications (nor of subjective justifications, but this might not be needed given that they are subjective). The question what the real logic of justification is, is open in our setting. Artemov's logic is a serious candidate.

9. Conclusion

A defect of the JTB-definition seems to be that it allows for justification that is too fragile, in the sense that it is not a justification any more if the agent is given some relevant information. We could also say that the justification is only a subjective justification, but is not a justification from an objective, omniscient point of view.

Based on the above idea we proposed our own solution, in a proof-theoretic fashion. We propose to incorporate both a notion of subjective and objective justification in the definition of knowledge. This solution takes into account the argument that JTB cannot be mended just by redefining the notion of justification as such to ensure that justification does not involve false lemma's: we really need to distinguish two separate kinds of justification. Further research could concentrate on how to refine the notions of subjective and objective justification and on what it means for them to match.

In this theory, what distinguishes knowledge from justified true belief is that the subjective justification matches an objective justification, where an objective justification is a justification from the perspective of an omniscient agent. An objective justification can be formalised as a sound derivation, which incorporates in the set of premisses all the relevant information.

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